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The calculation of flow fields by panel methods: a report on Euromech 75

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1. Introduction

Euromech Colloquium number 75 was arranged jointly by the European Mechanics Committee and the Fachausschuss für Numerische Methoden in der Strömungsmechanik of the Gesellschaft für Angewandte Mathematik und Mechanik (Committee for Numerical Methods in Fluid Mechanics of the Society for Applied Mathematics and Mechanics). The colloquium was held at Rhode near Braunschweig at the Management-Training Centre of the Volkswagenwerk Company from 10–13 May 1976. It was attended by 54 participants from nine countries. The organization was by H. Körner, DFVLR-Institut für Aerodynamik, Braunschweig, and E. H. Hirschel, DFVLR-Institut für Angewandte Gasdynamik, Köln. Twenty-eight papers were presented in four sessions.

Panel methods are used to solve inviscid linear subsonic and supersonic flow problems. The basis of these methods may be described for incompressible flow governed by the Laplace equation, which has to be solved for the unknown potential Φ. Once the potential Φ is known, the velocity, the pressure distribution and the overall forces on a body immersed in the flow can be evaluated. In the prediction case we have a Neumann problem with the boundary condition of tangential flow at the surface and the condition of uniform flow at infinity. The flow then may be separated into an undisturbed incident flow and a perturbation flow. The second flow field then has to be evaluated by a network technique, e.g. a finite-difference approach, but this is not of the ordinary type.

With the help of Green's theorem the Laplace equation can be transformed into an integral equation. This reduces the problem to the determination of the perturbation potential on the surface of the body alone. This perturbation potential has a singular behaviour. The transformation into an integral equation and the use of a distribution of singularities on the surface have led to the names 'integral-equation method' and 'singularity method' both referring to this class of methods.

Panel methods form a subset of these methods and are apparently the most advanced. The surface of the body is divided into a number of finite planar or non-planar 'panels' with an unknown singularity distribution and collocation points to satisfy the boundary conditions. The evaluation of the discrete singularity distribution leads to a large algebraic system which must be solved with the aid of a digital computer. Panel methods are widely used for the prediction of local and overall forces on wings, bodies and even whole aeroplanes. They can also be applied to vehicle and industrial aerodynamics and to problems in turbomachinery flow in cases where viscous effects are of minor importance.

In order to avoid confusion it should be stated that the colloquium dealt with 'panel methods', not with 'finite-element methods'. Finite-element methods are quite different and are based on a variational principle or on the Galerkin technique. The working plane of finite-element methods is the flow field itself, where a functional has to be minimized. In contrast to this, panel methods are in general applied on the surface of a body or a wing and, instead of a minimum principle to be satisfied, there is a Neumann or Dirichlet problem to be solved.

The four-day programme was divided into four sessions of different length, dealing with 'First-Order Surface Panel Methods', 'Higher Order Surface Panel Methods', 'Mean-Surface Panel Methods and Lattice Methods' and 'Field Panel Methods'. No publication of the papers presented is planned.

2. First-order surface panel methods

The subject of sessions 1 and 2 was surface panel methods. For this category of methods the singularity distribution is located on the actual surface of the body and the boundary conditions are satisfied on the actual surface too. The idea of distributing singularities on the surface was first used by Lotz (1931) and afterwards by Vandrey (1937), but these methods did not succeed since the computation was rather laborious by hand and digital computers were not available at that time.

The first session dealt with first-order panel methods, which are based on plane surface panels and a constant singularity distribution on each panel. This type of panel method, first developed by Smith & Hess (1967), is already a classical approach, but up to now, it has not been fully exploited.

The session started with a survey paper by J. L. Hess. He showed how far these methods have been developed since the first papers on this subject were published almost fifteen years ago. Today, complete aeroplane configurations including the tailplane, nacelles, external stores and so on can be calculated with high accuracy. After clearing up the difficulties connected with the implementation of the Kutta condition as well as formulation problems in the lifting case, some workers are now tackling the introduction of viscous effects either through an equivalent blowing at the surface or by adding a displacement thickness to the body contour. The first approach can be incorporated into the solution schemes more easily since the geometry does not change in this instance.

V. Losito gave a paper in which he discussed the use of a higher-order singularity distribution on the surface. This topic was discussed in greater detail later on, during the second session. In the next presentation, B. Hunt showed that conventional first-order methods with a source distribution on the surface and vortices inside the body lead to erroneous results when the wing thickness is decreased. He demonstrated that a revised form of the vortex-filament model, in which he used a piecewise-constant vorticity distribution proportional to the load distribution, gives better results for wings down to an aerofoil thickness of 2 %.

W. Kraus, from Messerschmidt, Bölkow & Blohm, reported on the work with panel methods which has been done at MBB during the last ten years. Methods have been both developed and adapted for subsonic and supersonic cases. Much has been done on topics such as wing–body–tail interference, the wing wake, external stores and so on. In addition, the methods have been extended to the prediction of store separation trajectories and ice development at the aerofoil nose.

S. R. Ahmed presented work on modelling separated regions when calculating flows around thick bodies like silos or even cars. If the separation region can be described sufficiently well, the calculated pressure compares well with the measured pressure in the non-separated region. A paper on the calculation of jet interference effects was presented by H. Struck of VFW-Fokker. He considered jet–body interference for a high-aspect-ratio STOL aircraft (turbo-engine mounted on a wing) and for a low-aspect-ratio VTOL aircraft (lifting jet). Apart from the difficulties in modelling the jet geometry and the jet entrainment, it has been shown that the treatment of such interference problems by potential theory is possible in principle. A problem similar to the VTOL interference problem has been treated, although much less thoroughly, by H. Schmitt, DFVLR-Göttingen. He studied the pressure distribution induced on a wall from which a jet emerges perpendicular to the mainstream.

A paper on the calculation of the flow in a cascade with contracting side walls was presented by J. Renken. The assumption of constant vorticity in the spanwise direction enabled him to devise a relatively simple panel method for the calculation of the three-dimensional flow in the cascade. The last talk in this session on first-order panel methods was by J. Steinheuer, who studied the passing of two high-speed trains using a two-dimensional quasi-steady model. The quasi-steady approach consists of solution of the Laplace equation at discrete positions of the trains and calculation of the pressure from the unsteady form of Bernoulli's equation (see Sockel 1971*a*, *b*).

Since the first-order surface panel method is well established, it has been used to tackle several problems in aerodynamics. But it seems to us that there is great danger in using it as a black-box method. Everybody who works with it should have a good feeling for the physical behaviour of the flow; otherwise the result of the method may be misleading.

3. Higher-order surface panel methods

These methods have been developed to overcome difficulties connected with the first-order approach such as leakage problems, inaccuracy of the velocity gradient for boundary-layer calculations and improper treatment of critical regions like the wing root and wing tip. In contrast to the first-order methods, the surface panels are no longer plane and the elementary singularity distribution on a panel can have a linear, parabolic or higher-order behaviour with spline fitting across the panel boundaries. One of the pioneers in the development of these methods is A. Roberts.

W. Schmidt from the Dornier Company opened the session with a survey paper which discussed in detail the main features of higher-order surface panel methods. A linear variation of the singularity strength is considered to be sufficient. The pay-off is seen in (*a*) insensitivity of the solution to panel size variations, (*b*) compatibility with automated input geometry generation, (*c*) minimal user experience, (*d*) a large range of applicability (analysis or design) and (*e*) avoidance of leakage problems in internal flow situations. E. F. F. Botta of the University of Groningen then presented a technique using cubic spline approximations both for a doublet distribution on the surface of the body and its wake and for the representation of the surface geometry. The calculations for aerofoils and the first results for three-dimensional wings show very high accuracy with surprisingly few panels. A. Roberts from the British Aircraft Corporation presented his 'spline–Neumann system', which is in use for flows past bodies and wing–body combinations, i.e. past real aircraft configurations. He sees his method, especially the representation of the surface geometry, as being most useful in connexion with boundary-layer calculations, where the geometry plays an important role. When these interconnexions are seen as part of a computer-aided design system the higher cost of the spline system compared with the linear panel system will loose its significance. Since Roberts' method is one of the most advanced higher-order methods which deals with bicubic functions for the singularities and the contour, it is widely used as a standard of accuracy for other methods.

A short report was presented by G. M. Weakley on the development and application of a higher-order panel method for the calculation of wing–pod interference effects at Rolls-Royce. P. E. Rubbert gave a talk on the application of a higher-order subsonic panel method to configurations with free vortex flow. These cases can be solved only with great difficulty using first-order panel methods. He discussed the three-dimensional flow about wing and wing–body combinations with leading-edge vortex separation, including effects of wing thickness, fuselage modelling and compressibility corrections. The vortex sheets are panelled and their location and form is calculated using nonlinear boundary conditions which require alignment with the local flow and support no pressure jump. Good results already have been obtained although the method is still under development.

The last talk in this session presented trial calculations made for selected problems by Boeing, BAC and NLR. This report was to have been given by R. C. Lock, but was actually given by B. Hunt since Dr Lock was not available. Since this work is not finished, the results allow no firm conclusions about the applicability of the methods. Some properties of panel methods discussed earlier showed up clearly, pointing the way to possible improvements. Other firms and research establishments are invited to participate. The results of these calculations will be collected by NLR and published later.

Some conclusions and recommendations from this session may be given:

Although some improvements to the classical first-order method are still possible, the need for more accurate solutions is obvious and requires higher-order techniques.

Higher-order methods may work with a reduced number of panels. Nevertheless, more computer time is generally required than for first-order methods because the evaluation of the influence coefficients is rather laborious.

It seems worth while to check whether it is favourable to redefine the boundary conditions as proposed by Rubbert. When using a vortex or doublet distribution on the surface, the problem is well posed if the inner flow is considered to be zero. This condition has already been applied by Prager (1928) and Martensen (1959).

4. Mean-surface panel methods and lattice methods

In this session methods were presented for the case where the singularity distribution is located on the chordal or mean surface of the wing. As a first approximation the boundary conditions are also satisfied on the mean surface. This is the classical approach of lifting-surface theory and the first methods which were developed dealt with this problem. The discretization most often used is the vortex-lattice technique. An early approach has been described by Falkner (1943), but the first widely used method was developed by Rubbert (1962); Hedman (1966), Giesing (1968) and Körner (1972) then followed the same lines. Woodward (1968) extended this method to thick wings and, after some modification, applied it to linear supersonic flow.

The first three papers of this session dealt with the classical wing problem. In the first paper, given by A. L. Gustavsson of the FFA, improvements to the Woodward method were reported. These were obtained mainly by satisfying the boundary condition on the wing surface and on the actual body. A further improvement was achieved by replacing the insufficient vortex representation of the body by a source system. In this way significant improvements were obtained for thick wings and for complicated axisymmetric bodies. C. C. L. Sells, RAE, presented a method for iterative calculation of subcritical flow around thick cambered wings. As a singularity representation he used a distribution of sources and doublets on the mean chordal surface. In order to treat thickness–lift interference properly he used an iterative procedure. The method may be used for prediction as well as for design purposes. A source-lattice method for wing-thickness design was presented by J. M. J. Fray, NLR.

The next five papers showed that the vortex-lattice approach is a fairly universal technique which can be applied to different flow problems. The state of the art for the vortex-lattice technique for total-force prediction was discussed by C. W. Lucchi, Dornier Company. He considered these methods to have a high degree of accuracy and speed owing to improvement of both the theoretical model and the numerical implementation. In his paper Lucchi discussed the most important characteristics of such methods and showed by selected results their scope and applicability.

B. Maskew discussed his subvortex technique for the calculation of flow fields where a discretized vortex sheet is approached closely. The objective is to calculate velocities at arbitrary points, not just at the midpoint between vortices. This was achieved by splitting a vortex into an increasing number of subvortices when it is approached. The method can be extended to three dimensions and should improve nonlinear methods for the calculation of interference problems, including force-free wakes.

A vortex-lattice method was applied to study interference effects between swept wings and tunnel walls by I. A. Lind, Volvo. Although some problems concerning the method will have to be solved, very useful results have been obtained so far. Of particular interest were the results he obtained for the pressure distribution on the wall of a wind tunnel with a closed test section. The pressure distribution influences the boundary-layer development on the tunnel walls, which in turn influences the flow on the model.

D. Hummes, DFVLR, presented a vortex theory for calculating rotor–wing interference on helicopters. The main difficulty in this type of problem is the modelling of the free vortices originating from the tips of the rotor blades. These vortices undergo strong deformations in the region behind the helicopter and finally form a rolled-up vortex sheet similar to that behind an aeroplane. The geometry of this flow field is extremely complicated, so that far-reaching simplifications had to be introduced in order to arrive at a practical computation method. The tracking of a single free vortex and the mechanism which causes a detached vortex sheet to roll up were discussed by S. A. Jepps.

Two talks on unsteady problems concluded the session. U. May presented work on a vortex-panel method for slowly oscillating lifting surfaces at subsonic speeds. The method is similar to Woodward's and is based on a low-frequency approximation of the kernel of Küssner's integral equation. R. Roos gave a survey of results obtained at the NLR using two panel methods to calculate the aerodynamic loading on oscillating aeroplane configurations at subsonic speeds. The methods are a doublet-lattice method for thin lifting surfaces and the NLR panel method for unsteady flow past wing–body combinations. Finally a technique for the introduction of the local Mach number correction into the first method was discussed.

Summarizing this session, it may be said that the mean-surface panel methods can easily be used for several problems in steady and unsteady aerodynamics. Thanks to their simplicity they are easy to manage and the calculation time is, in general, less than with surface panel methods.

5. Field panel methods

The last session dealt with a rather new category of methods: the so-called field panel methods. In contrast to what was said initially, these methods deal with nonlinear flow equations and the panel mesh covers the whole flow field.

Following an idea of Oswatitsch (1950), plane nonlinear subsonic flow, which is governed by a Poisson equation, may be treated as an incompressible flow on which is superimposed a flow field created by compressibility sources throughout the flow field whose strength is equivalent to the term on the right-hand side of the Poisson equation. R. Stricker from MBB presented a paper on a method for the calculation of subsonic flows about aerofoils of arbitrary cross-sectional shape. The contour of the aerofoil is replaced by a continuous vortex distribution and compressibility is simulated by a continuous source distribution throughout the flow field. The resulting set of equations is solved after introducing a panel-type discretization of the contour and the compressibility source field. The results compare well with experimental data. Extension to transonic and three-dimensional flow seems possible but some difficulties will arise. Mme G. Coulmy from CNRS, France, gave a talk on the extension of the method involving a discrete distribution of singularities to the computation of a velocity field with non-zero divergence or curl. The method is intended for the treatment of both compressibility and viscous effects in flows around bodies. Many examples were given which illustrated the applicability of the method. In the last paper, by J. Maczynski from Poland, approximation by global functions (as distinct from functions given pointwise as in finite-difference methods and local functions as in finite-element methods) was discussed. Such methods can be very simple to manipulate and are suited in some cases to the investigation of linear and quasi-linear problems. Another advantage is the possibility of investigating accuracy limitations, existence and uniqueness easily.

After this session it was asked whether field-panel methods are a good way to overcome the difficulties connected with the treatment of subsonic–supersonic regions in transonic flow. The results presented showed that the system is stable for mixed-flow regions but Stricker mentioned that his approach is strictly valid for subcritical flow only. In a proper approach to the mixed-flow problem, ranges of dependence have to be arranged in the supersonic region and the treatment of the shock wave must be examined.

Mme Coulmy's concept of evaluating the viscous flow around an aerofoil seems to be an interesting alternative to the boundary-layer-displacement concept and the suction concept. Especially in the treatment of the trailing edge, this idea may lead to improved results.

6. Conclusions

The colloquium has shown the advancement and refinement of current panel methods for the calculation of potential flow fields. In the general discussion it became clear that present-day computer-aided design systems, even if they contain only a panel method together with just the geometry model, are extremely complicated and require a large amount of maintenance. The geometrical input and the discretization still pose serious problems. The discretization problem can be overcome by introducing a higher-order surface representation and consequently automatic generation routines. Methods for the design problem do not seem to be so far advanced as methods for direct problems, especially for thick configurations. The treatment of free vortex sheets and free vortices has already made much progress. Problems involving jet interference are still handicapped by the fact that not enough is known about entrainment properties and jet deformation in cross-flow. Considering the fact that some papers already discussed the inclusion of boundary-layer effects via coupling of panel methods with boundary-layer methods, one wonders why there is so little research on understanding and describing three-dimensional turbulent boundary layers on wings and bodies.

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